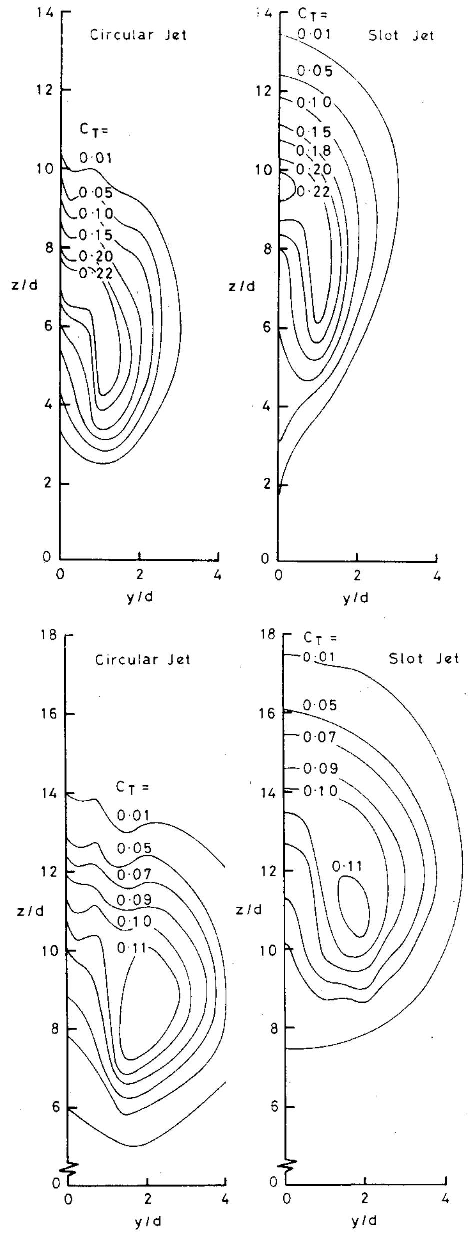

Fig. 2 Contours of C_T in y-z plane, circular and slot jet, $\lambda = 8$: a) $x/d = 2.0$; b) $x/d = 7.5$.

different downstream locations. Similar results for the circular jet are provided for comparison. It is seen that the slot jet has a distinct thin region behind the jet near the surface of the flat plate. Temperature contours in the plane of symmetry indicate that the mixing, at least near the plane of symmetry, is more intense in the region just downstream of the slot jet than it is for the circular jet. This thin region behind the slot jet persists at least to $x/d = 3.5$, and disappears by $x/d = 7.5$ (Fig. 2b). By $x/d = 7.5$ the shape of the slot jet in the y-z plane is not very different from that of the circular jet.

Concluding Remarks

The flowfield associated with a slot jet in crossflow differs in several respects from that of a round jet. Immediately downstream of the exit, the mixing of the jet and freestream fluid is much more pronounced for the slot jet than for the round jet. Also, the relative positions of the temperature and pressure centerlines are different for the two jets.

References

[1] Jordinson, R., "Flow in a Jet Directed Normal to the Wind," British Aeronautical Research Council, R & M 3074, Oct. 1956.

[2] Pratte, B. D. and Baines, W. D., "Profiles of the Round Turbulent Jet in a Crossflow," *Journal of the Hydraulics Division, Proceedings of the American Society of Chemical Engineers,* Vol. 92, No. HY6, Nov. 1967, pp. 53-64.

[3] Fearn, R., and Weston, R.P., "Vorticity Associated with a Jet in a Crossflow," *AIAA Journal,* Vol. 12, Dec. 1974, pp. 1666-1671.

[4] Maussa, Z. M., Trischka, J. W., and Eskinazi, S., "The Near Field in the Mixing of a Round Jet with a Cross-Stream," *Journal of Fluid Mechanics,* Vol. 80, Pt. 1, April 1977, pp. 49-80.

[5] Antani, D. L., "An Experimental Investigation of the Vortices and the Wake Associated with a Jet in Crossflow," Ph.D. Thesis, Georgia Institute of Technology, Atlanta, Ga., Aug. 1977.

[6] Mosher, D. K., "An Experimental Investigation of a Turbulent Jet in Crossflow," Ph.D. Thesis, Georgia Institute of Technology, Atlanta, Ga., Dec. 1970.

[7] Ruggeri, R., Callaghan, E., and Bowden, D., "Penetration of Air Jets Issuing from Circular, Square, and Elliptical Orifices Directed Perpendicularly to an Air Stream," NACA TN 2019, Feb. 1950.

[8] Reilly, R. S., "Investigation of the Deformation and Penetration of a Turbulent, Subsonic Jet Issuing Transversely Into a Uniform Subsonic Main Stream," Ph.D. Thesis, Univ. of Maryland, College Park, Md., 1968.

[9] Kamotani, Y. and Greber, I., "Experiments on a Turbulent Jet in a Cross-Flow," Case Western Reserve University, Cleveland, Ohio, FTAS/TR-71-62, June 1971; also *AIAA Journal,* Vol. 10, Nov. 1972, pp. 1425-1429.

Behavior of Vortex Wakes from Oscillating Airfoils

J. Katz* and D. Weihs†

Technion – Israel Institute of Technology, Halfa, Israel

Introduction

THE vortex wake of an oscillating airfoil has long been studied,[1] and it is now well known that under certain conditions strong nonlinear interactions can develop.[2,3] This appears in the form of agglomerations of vortex elements resulting in discrete concentrations of vorticity which can then be analyzed as separate vortices.[4]

In this Note, we present a simple and rapid numerical technique for calculation of the time-dependent growth and evolution of wakes. The parameters influencing the rollup are examined and the ranges of reduced frequencies for which such instabilities occur are determined.

Analysis

A thin airfoil moving along a curved path in an incompressible inviscid fluid is analyzed. Pitching and heaving motions, as well as finite rigidity of the foil can cause displacement $h(x,t)$ of points on the foil from the average trajectory (Fig. 1). This motion is limited in the present analysis to cases where the local angle of attack including effects of angular motion ω is always in the linear part of the lift slope curve, and the displacements are such that $h/c \ll 1$ (where c is the chord length).

The velocity potential ϕ of the flow can be written, and solved, in two parts – as a result of the linearizing limitations above. Thus the disturbance potential due to the foil motion ϕ_0 is separated from the wake potential ϕ_w.

In order to deal with arbitrary, nonsteady planar motions of the foil, the wake potential ϕ_w is given in the form of a row of adjacent discrete vortices (Fig. 1). At any given time the

Received Aug. 22, 1978.

Index category: Nonsteady Aerodynamics.

*Research Associate, Department of Aeronautical Engineering.

†Associate Professor, Department of Aeronautical Engineering.

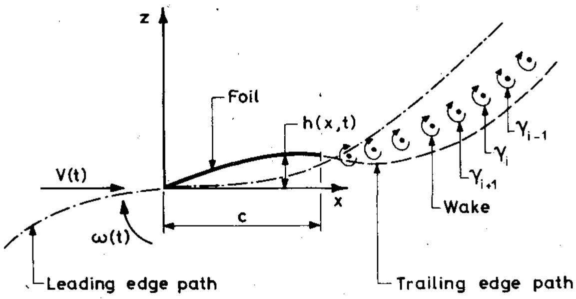

Fig. 1 Schematic description of model.

strengths of the wake vortices (or ϕ_w) are known from previous steps except for the latest vortex γ_n – which is defined as the vorticity shed during the latest time step. The strength of this vortex is determined by Kelvin's Theorem:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t}=\frac{\mathrm{d}\Gamma_f}{\mathrm{d}t}+\frac{\mathrm{d}\left(\sum_{i=1}^{n}\gamma_i\right)}{\mathrm{d}t}=0 \tag{1}$$

where Γ_f is the instantaneous circulation of the wing and γ_1, γ_2, ... γ_n are the wake elements.

Further simplification of the nonsteady problem is obtained by its solution in a coordinate system attached to the foil's leading edge, where the x direction is locally tangent to the flight path.[5] Here the only unknown is the latest wake vortice γ_n and the foil circulation which is found by a two-dimensional analytical technique described below. The above method considerably simplifies the numerical solution of such problems, compared to "fully numerical" techniques such as doublet lattices, or vortex lattice methods.[3,6]

The boundary condition for the downwash W is stated now on the foil surface $z=h(x,t)$ where $V(t)$ stands for the in-

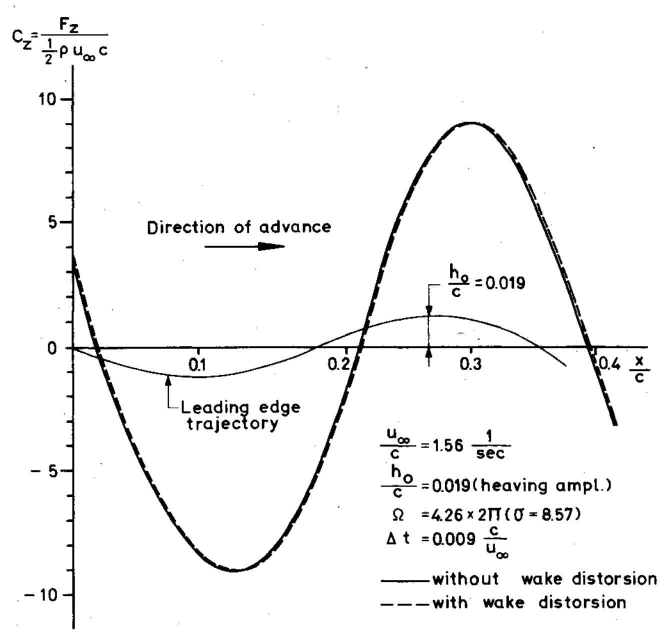

Fig. 3 Influence of wake distortion on the normal force coefficient in oscillatory motion.

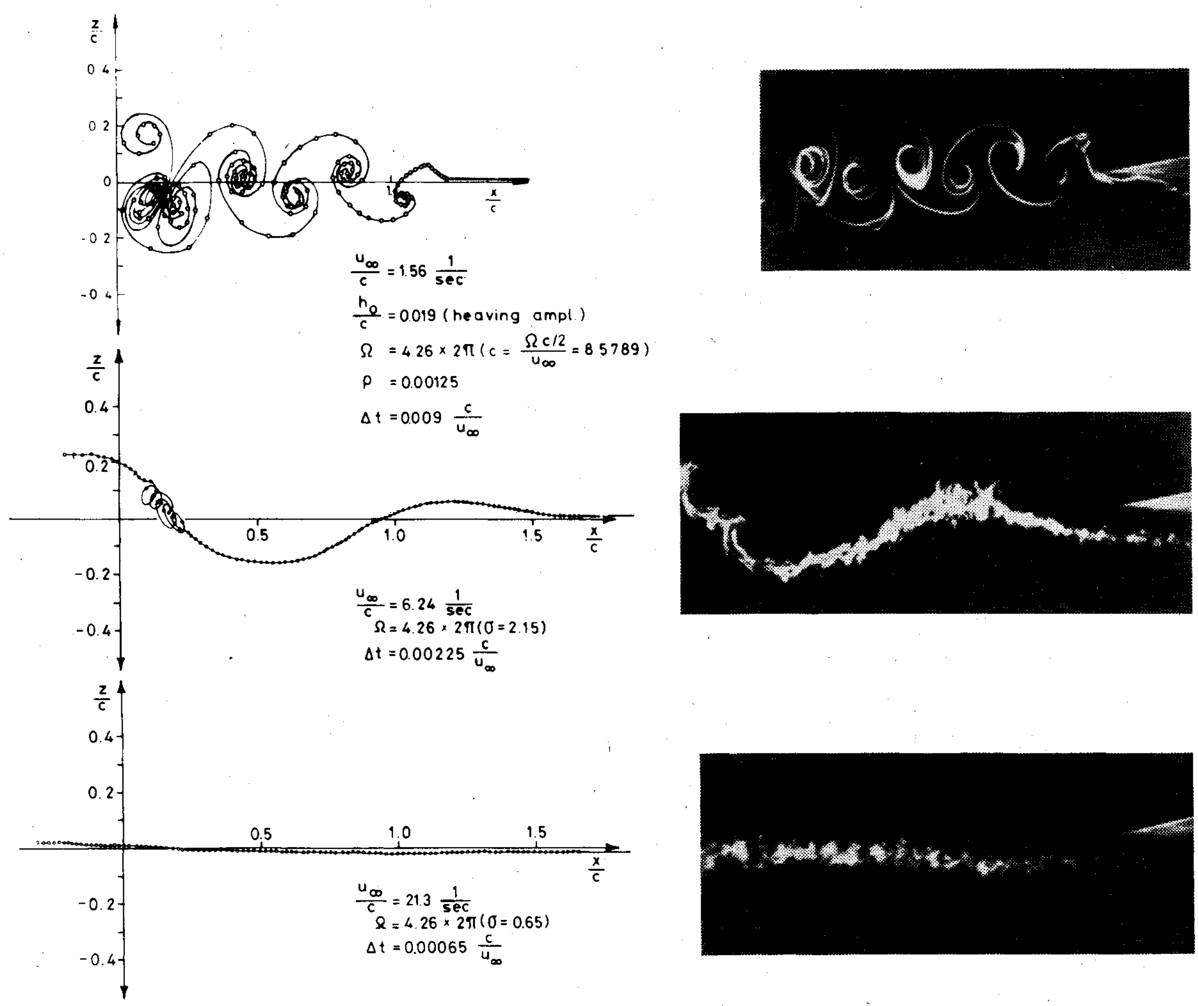

Fig. 2 Comparison of experimental and calculated wake patterns for various reduced frequencies.

stantaneous flight velocity and

$$\frac{\partial\phi_0}{\partial z}\Big|_{z=0} = V(t)\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} - \frac{\partial\phi_w}{\partial z} - \omega\cdot x \equiv W \qquad (2)$$

The solution for the foil vorticity distribution γ, assuming the existence of the Kutta condition, is analogous to classical wing theory[7]

$$\gamma(\theta,t) = 2V(t)\left[A_0(t)\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n^{(t)}\sin n\theta\right] \qquad (3)$$

where the chordwise coordinate is $x=(c/2)(1-\cos\theta)$, and the Fourier coefficients A_n are found as

$$A_0 = \frac{-1}{\pi}\int_0^{\pi}\frac{W}{V}\mathrm{d}\theta \qquad A_n = \frac{2}{\pi}\int_0^{\pi}\frac{W}{V}\cos n\theta\,\mathrm{d}\theta \qquad (4)$$

The momentary lift L is found by integrating the pressure difference along the foil:

$$\begin{aligned} L(t) &= 2\rho\int_0^c\left[\frac{\partial\phi_0}{\partial t} + V(t)\frac{\partial\phi_0}{\partial x}\right]\mathrm{d}x \\ &= \pi\rho c\Big\{\Big[A_0V^2 + \frac{3c}{4}\frac{\mathrm{d}}{\mathrm{d}t}(A_0V)\Big] \\ &\quad + \Big[\frac{1}{2}A_1V^2 + \frac{c}{4}\frac{\mathrm{d}}{\mathrm{d}t}(A_1V)\Big] + \Big[\frac{c}{8}\frac{\mathrm{d}}{\mathrm{d}t}(A_2V)\Big]\Big\} \qquad (5) \end{aligned}$$

where $\partial\phi_0/\partial x$ on the foil is $\gamma/2$ and

$$\frac{\partial\phi}{\partial t} = \frac{\partial}{\partial t}\int_0^x\frac{\gamma}{2}\mathrm{d}x$$

Results and Discussion

The rollup of the wake of an oscillating airfoil in heaving motion is shown in Fig. 2. The calculated vortex configuration is practically identical to Bratt's[2] experimental results, proving the validity of the present method. Similar results were obtained by Giesing[3] and Djojodihardjo and Widnall,[6] using the doublet lattice method. The present calculation of the wake vortex sheet was done using much shorter time intervals, enabling fuller study of the details of the rolling up process, while requiring much shorter calculation times. Comparison of the time-dependent variation of lift on the foil including the deforming wake, with results of a calculation where the wake was not allowed to deform (Fig. 3), shows no significant difference. This is due to the fact that the wake deformation increases with the distance behind the foil, in a symmetric manner. Therfore the overall wake influence of the momentary lift is not seriously affected.

Figure 2 indicates that vortex sheet breakup occurs within the first period when the reduced frequencies are larger than a certain critical value:

$$\sigma = \frac{\omega c/2}{V} > 2$$

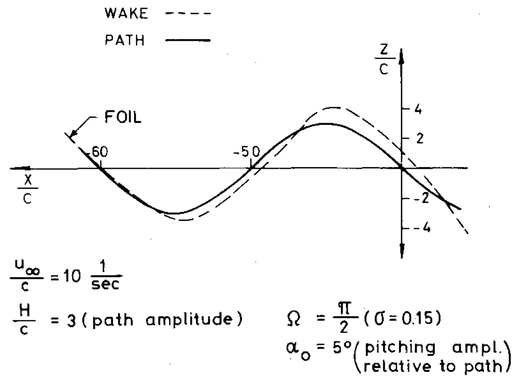

Fig. 4 Wake distortion in large amplitude oscillating motion.

This breakup prediction proved to be in excellent agreement with Bratt's[2] experimental results. The effect of such breakup and agglomeration on the lift calculation was negligible, thus justifying simplified point vortex models of the wake[4] of oscillating airfoils. When considering large amplitude trajectories (Fig. 4) the small disturbance restriction limits the reduced frequency. In these cases vortex rollup does not take place in the vicinity of the foil (Fig. 4), and the wake follows the trailing-edge trajectory.

In conclusion, this Note presents a rapid, efficient method for calculating wake configurations (the calculations for Fig. 2 took 4 s of CPU time on an IBM 370/168 computer), showing that for lift calculations one can almost always take the wake to stay on the trailing-edge trajectory. The rollup of the wake and its breakup into discrete vortices was shown to be dependent on the reduced frequency.

References

[1] Fage, A. and Johansen, F.C., "The Structure of the Vortex Sheet," *Philosophical Magazine,* Vol. 5, 1928, p. 417.

[2] Bratt, J.B., "Flow Patterns in the Wake of an Oscillating Airfoil," Aeronautical Research Council, R&M 2773, 1953.

[3] Giesing, J.P., "Nonlinear Two-Dimensional Unsteady Potential Flow with Lift," *Journal of Aircraft,* Vol. 5, March-April 1968, pp. 135-143.

[4] Weihs, D., "Semi-infinite Vortex Trails, and their Relation to Oscillating Airfoils," *Journal of Fluid Mechanics,* Vol. 54, Pt. 4, Aug. 1972, pp. 679-690.

[5] Katz, J. and Weihs, D., "Hydrodynamic Propulsion by Large Amplitude Oscillation of an Airfoil with Chordwise Flexibility," *Journal of Fluid Mechanics,* Vol. 88, Pt. 3, Oct. 1978, pp. 485-497.

[6] Djojodihardjo, R.H. and Widnall, S.E., "A Numerical Method for the Calculation of Nonlinear, Unsteady Lifting Potential Flow Problems," *AIAA Journal,* Vol. 7, Oct. 1969, pp. 2001-2009.

[7] Robinson, A. and Laurmann, J.A., *Wing Theory,* Cambridge University Press, 1956, Chap. 2.

Announcement: 1978 Author and Subject Index

The indexes of the six AIAA archive journals (*AIAA Journal, Journal of Aircraft, Journal of Energy, Journal of Guidance and Control, Journal of Hydronautics, Journal of Spacecraft and Rockets*) will be combined and mailed separately early in 1979. In addition, papers appearing in volumes of the *Progress in Astronautics and Aeronautics* book series published in 1978 will be included. Librarians will receive one copy of the index for each subscription which they have. Any AIAA member who subscribes to one or more Journals will receive one index. Additional copies may be purchased by anyone, at $10 per copy, from the Circulation Department, AIAA, Room 730, 1290 Avenue of the Americas, New York, New York 10019. **Remittance must accompany the order.**

Ruth F. Bryans
Administrator, Scientific Publications